Eur. Phys. J. A **15**, 223–227 (2002)
DOI 10.1140/epja/i2001-10258-0

THE EUROPEAN PHYSICAL JOURNAL A

Superallowed $0^+ \rightarrow 0^+$ beta-decay and CKM unitarity

Recent results and future prospects

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Received: 21 March 2002 /
Published online: 31 October 2002 –

Abstract. Precise measurements of the ft-values for superallowed nuclear β-decay yield results that support conservation of the weak vector current but violate the unitarity of the Cabibbo-Kobayashi-Maskawa matrix by more than two standard deviations. This apparent violation of the Standard Model has led to considerable theoretical and experimental activity largely focussed on reducing uncertainties in the small correction terms that account for charge-dependent nuclear effects. This activity is outlined and the prospects for future sharpening of the unitarity test are assessed.

PACS. 23.40.Bw Weak-interaction and lepton (including neutrino) aspects – 23.40.Hc Relation with nuclear matrix elements and nuclear structure

1 Introduction

One of the most exacting tests of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is provided by nuclear β-decay. Precise measurements of β-decay transitions between $T = 1$ analog 0^+ states are used to determine G_V, the vector coupling constant; this, in turn, yields V_{ud}, the up-down element of the CKM matrix. To date, the ft-values for nine $0^+ \rightarrow 0^+$ transitions have been determined to a precision of $\sim 0.1\%$ or better; these span a wide range of nuclear masses from ^{10}C, the lightest parent, to ^{54}Co, the heaviest. As anticipated by the Conserved Vector Current hypothesis, CVC, all nine yield consistent values for G_V, but the value of V_{ud} derived from their average yields a more provocative result. The unitarity test of the CKM matrix, which is made possible by this precise value of V_{ud}, fails by more than two standard deviations [1] viz

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9968 \pm 0.0014. \quad (1)$$

This result would have far-reaching consequences on the Standard Model if it were to be confirmed with improved statistical definition.

The potential impact of such an outcome has led to considerable recent activity, both experimental and theoretical, in studying superallowed $0^+ \rightarrow 0^+$ transitions, with special attention being focussed on the small correction terms that must be applied to the experimental ft-values in order to extract G_V. Specifically, G_V is obtained from each ft-value via the relationship [1]

$$\mathcal{F}t \equiv ft(1+\delta_R)(1-\delta_C) = \frac{K}{2G_V^2(1+\Delta_R^V)}, \quad (2)$$

with

$$K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2/(m_e c^2)^5 = (8120.271 \pm 0.012) \times 10^{-10}\,\mathrm{GeV}^{-4}\mathrm{s}, \quad (3)$$

where f is the statistical rate function, t is the partial half-life for the transition, δ_C is the isospin-symmetry-breaking correction, δ_R is the transition-dependent part of the radiative correction and Δ_R^V is the transition-independent part. Here we have also defined $\mathcal{F}t$ as the "corrected" ft-value.

It is now convenient to separate the radiative correction into two terms:

$$\delta_R = \delta'_R + \delta_{NS}, \quad (4)$$

where the first term, δ'_R, is a function of the electron's energy and the charge of the daughter nucleus, Z; it, therefore, depends on the particular nuclear decay, but is *independent* of nuclear structure. The second term, δ_{NS}, accounts for transitions in which the axial-vector interaction flips one nucleon's spin and the electromagnetic interaction flips that of another nucleon; these together result in no net nuclear spin change. Like δ_C, this term clearly depends on the details of nuclear structure. To emphasize the different sensitivities of the correction terms we re-write the expression for $\mathcal{F}t$ as

$$\mathcal{F}t \equiv ft\big(1+\delta'_R\big)\big(1+\delta_{NS}-\delta_C\big), \quad (5)$$

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where the first correction in brackets is independent of nuclear structure, while the second incorporates the structure-dependent terms.

The term δ'_R is calculated from standard QED, and is currently evaluated to order $Z\alpha^2$ and estimated in order $Z^2\alpha^3$ [2,3]; its values are around 1.4% and can be considered very reliable. Calculations for δ_{NS} [4] and for δ_C (see, for example, refs. [5,6]) have been made based on the nuclear shell model. Nuclei with $10 \leq A \leq 54$, particularly those with $A < 40$, are generally well described by the shell model, and these calculations of the correction terms have been carefully linked to other related observables such as the neutron and proton binding energies, the c-coefficient in the Isobaric Multiplet Mass Equation, and the non-analog $0^+ \rightarrow 0^+$ transition rates. The calculations and their constraints are described in more detail in ref. [1], where it is also shown that several independent calculations of δ_C are in good agreement with one another. Calculated values for δ_C range from 0.2 to 0.7% depending on the nucleus involved, and those for δ_{NS} from +0.03 to −0.4%. Given the success of the shell model in this mass region, and the agreement among independent calculations of δ_C, these results should also be rather reliable. Nevertheless, their uncertainties —deemed to be of order 0.1% (*i.e.* ∼ 10% of their own value)— are major contributors to the overall uncertainty on the unitarity test.

As their input to the unitarity test of eq. (1), the nuclear decays yield the result $V_{ud} = 0.9740 \pm 0.0005$. The contributions to this uncertainty are 0.0001 from experiment, 0.0001 from δ'_R, 0.0003 from δ_C and δ_{NS}, and 0.0004 from Δ_R^V. If the unitarity test is to be sharpened, then a significant improvement in the precision of Δ_R^V is clearly the most pressing objective, but this is a purely theoretical problem and, in fact, one that is currently being addressed. That leaves the nuclear-structure–dependent corrections as the most important area where nuclear experiments can play a critical role. There is considerable activity, both experimental and theoretical, now underway in probing these nuclear corrections with a view to reducing the uncertainty they introduce into the unitarity test. In what follows, we describe the new results so far available and assess the prospects for significant advances in the near future.

2 Current and future directions

To measure an ft-value, three quantities are required from experiment: the transition energy, Q_{EC}, which is used in calculating f; the half-life, $t_{1/2}$, of the parent nuclide and the branching ratio for the superallowed transition, which together yield the partial half-life, t. There are two ways in which improvements in the unitarity test can be sought experimentally: through increased precision in measurements of the 27 quantities now used for the nine well-known superallowed transitions; or through new measurements for superallowed transitions not previously studied with sufficient precision. Needless to say, the nine include all the "easy" cases, in which the daughter nuclei are stable. With both parent and daughter being unstable, any new cases are necessarily more difficult to measure, as well as being more difficult to produce, thus limiting their statistical definition. Furthermore, they present unique new challenges not previously faced in such precision measurements. The most accessible new cases fall into two categories: those having even-even, $T_Z = -1$ parents with $18 \leq A \leq 42$ (nuclides such as ^{22}Mg, ^{30}S and ^{34}Ar); and those with odd-odd, $T_Z = 0$ parents with $A \geq 62$ (nuclides such as ^{62}Ga and ^{74}Rb).

Since the goal of any new measurement must be to test and constrain the calculated structure-dependent corrections, an important first step is to have a set of consistent calculations that apply both to the nine well-known transitions and to the new cases yet to be studied. New calculations of δ_C and δ_{NS} have just recently been completed [7], in which consistent model spaces and approximations have been used for both correction terms and for a large repertoire of superallowed transitions, new and old. These will provide a consistent standard for experimental comparison.

2.1 Improvements to existing results

The nine superallowed transitions already described have been the subject of intense scrutiny for at least the past three decades. Given the large number of careful, independent measurements, it is unlikely that further measurements will resolve the unitarity discrepancy by changing significantly the final value of V_{ud}. However, reduced experimental uncertainties on some of the measurements could heighten our confidence in the structure-dependent corrections. The point is illustrated in fig. 1, which shows the experimental ft-values and corrected $\mathcal{F}t$-values for all nine. Evidently, in these cases, at the current level of precision the nucleus-dependent corrections act very well to remove the "scatter" that is apparent in the experimental ft-values and effectively absent from the corrected $\mathcal{F}t$-values. Already, to a certain extent, this validates the calculated corrections, providing we accept CVC in the first place, but it does not do so with sufficient precision. For these cases, the maximum range of the scatter in ft-values is only 0.5%. With the experimental ft-value uncertainties at the 0.1% level, this only validates the corrections to about 20% of their value.

Improved precision on the experimental ft-values would naturally make this a more demanding control on the calculated corrections, but unfortunately an improvement of more than a factor of two in the uncertainties would require an experimental breakthrough that no one anticipates. Nevertheless, a careful look at the error budget for each ft-value measurement (see, for example, refs. [8,9]) does offer some opportunities for improvement even with currently available techniques. The Q-values of ^{10}C, ^{14}O, ^{26m}Al and ^{46}V, the half-lives of ^{10}C, ^{34}Cl and ^{38m}K, and the branching ratio for ^{10}C can all bear improvement. Though valuable, however, such improvements in old measurements would not act to constrain the calculated corrections as tightly as other new measurements on entirely different nuclides. These measurements, which

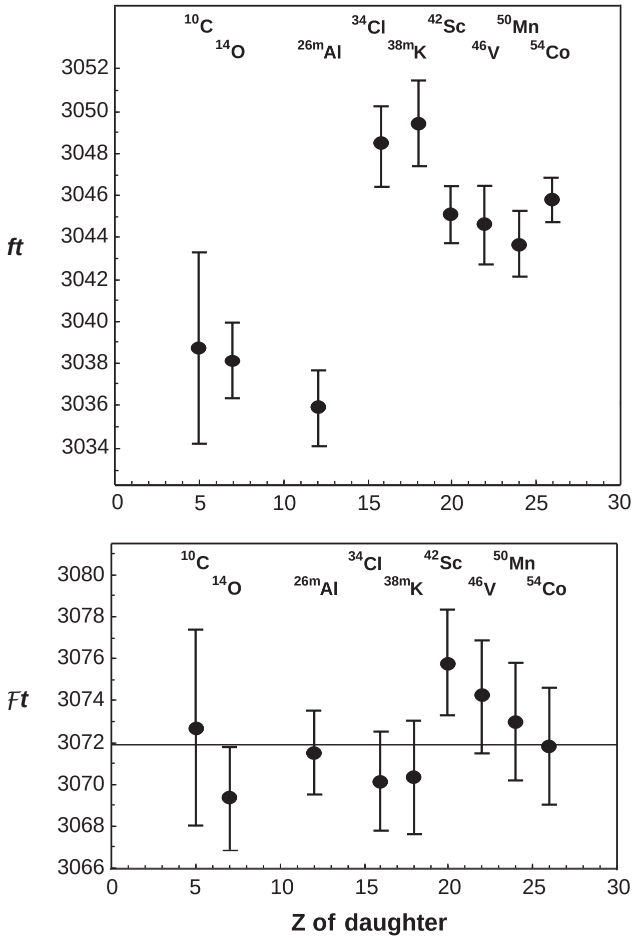

Fig. 1. Comparison of experimental ft-values and the corrected $\mathcal{F}t$-values for the nine well-known superallowed transitions. This illustrates the effect of the calculated nucleus-dependent corrections, which change from transition to transition. (The effect of δ_R' is virtually the same for all cases.)

are only just now becoming possible, also involve $0^+ \to 0^+$ decays but of nuclides for which much larger structure-dependent corrections are anticipated.

2.2 New $0^+ \to 0^+$ decays with $18 \leq A \leq 42$

As shown in eq. (5), the total effect of structure-dependent corrections on a superallowed transition is given by the difference $(\delta_C - \delta_{NS})$. For the nine well-known transitions, the values [7] of $(\delta_C - \delta_{NS})$ range from 0.25 to 0.77%, a total spread of 0.5%. If we now can incorporate the decays of $0^+, T_Z = -1$ nuclei with $18 \leq A \leq 42$, the range of values for $\delta_C - \delta_{NS}$ can be extended up to 1.12%, increasing the overall spread by nearly a factor of two, to 0.9%. If ft-values for these new transitions can be measured to $\pm 0.1\%$, the current standard, then a successful comparison of the type illustrated in fig. 1 could validate the structure-dependent corrections to 10% of their value, also a factor of two improvement over what is now possible. Most important, the new cases with $18 \leq A \leq 42$ all lie within the s-d and $f_{7/2}$ shells, like most of the nine cases upon which the unitarity test is currently based: *i.e.* the $(\delta_C - \delta_{NS})$ values soon to be tested by these new measurements have all been calculated with exactly the same shell model spaces as those currently being used. Any conclusions about the calculated corrections that result from these experiments will have direct and immediate impact on the unitarity test itself.

An experimental program is now underway at Texas A&M University to measure the half-lives and branching ratios for superallowed transitions from a number of $T_Z = -1$ nuclei in this mass region. This work has begun with ^{22}Mg (see refs. [10,11] for preliminary results) and will continue soon with ^{34}Ar. An independent measurement of the ^{34}Ar decay will also be made at the TRIUMF-ISAC facility. In both these cases, and in any others in this series to follow, a precise measurement of the corresponding Q_{EC}-values will also be required before an ft-value can be obtained. Though challenging —a precision of a few hundred electron volts is required— mass measurements of the nuclei involved are technically feasible. In fact, a suitable result for the ^{34}Ar mass has been presented at this conference by the ISOLTRAP collaboration [12].

The major experimental challenge with this class of superallowed decays arises from the fact that the daughter nuclei are odd-odd. This means that there are a number of 1^+ states in each daughter, which are populated by Gamow-Teller β-branches that compete strongly with the superallowed branch itself. Since all these branching ratios must be determined from the measured intensities of β-delayed γ-rays, a precision of $\sim 0.1\%$ for the superallowed branch often demands that the efficiency of the γ-ray detector must be known over the whole energy range to $\sim 0.1\%$ as well. This is an extremely demanding requirement, but one that has now been achieved [13]. Spectra from thirteen independent sources of ten different radionuclides, including one ^{60}Co source whose activity was known [14] to better than 0.1%, were recorded with an HPGe detector at Texas A&M and the results compared with *un-renormalized* Monte Carlo calculations performed with the CYLTRAN code [15] using the actual dimensions of the detector, which were determined, in most cases, by independent measurements. The agreement is excellent, quite sufficient for the detector efficiency to be quoted to 0.2% or better at all energies between 50 and 1836 keV.

2.3 New $0^+ \to 0^+$ decays with $A \geq 62$

The superallowed transitions from $T_Z = 0$ nuclei with $A \geq 62$ have one very positive feature and several rather negative ones. On the positive side, their corrections, $(\delta_C - \delta_{NS})$, take on very large values —between 1.4 and 1.7% [7]— which make them particularly appealing as test beds for the calculations. However, they have concomitant disadvantages too. First, the parent nuclides have very short half-lives (< 120 ms); this limits the precision with which their masses can be measured by current technology. Second, the nuclides lie in a region of rapid shape changes, making model-based calculations far less reliable

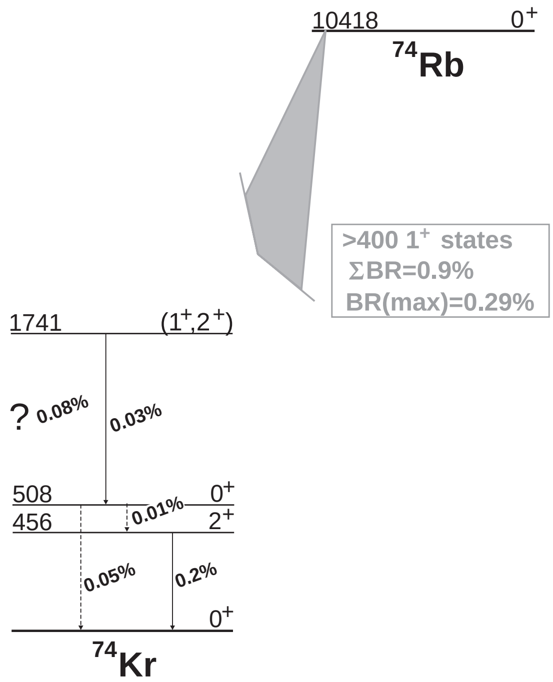

Fig. 2. β-decay scheme for ^{74}Rb showing the observed γ-rays (solid arrows) and conversion electrons (dashed arrows) in ^{74}Kr, together with their measured intensities relative to the total decays of ^{74}Rb. The number next to the question mark represents the total intensity of unplaced γ-rays. The results of shell model calculations for 1^+ states appear in the grey box.

than in the s-d and $f_{7/2}$ shells; any test of structure-dependent corrections in this region, successful or otherwise, would likely reflect more on the validity of the local nuclear model than on the correctness of calculations in any other region.

Finally, there is a difficulty inherent to all nuclei with high β-decay Q-values that lie in this mass region or heavier: there are many 1^+ states in the daughter nucleus, in addition to the 0^+ analog state, that can be populated by β-decay and, furthermore, though the branching ratios are individually very small, their total can be quite significant. Thus, while the total is required to determine the superallowed branching ratio precisely, many of the component branches are too weak to observe. This phenomenon was first pointed out more than two decades ago in calculations for a fictional nucleus named Pandemonium [16]. The impact on the nuclei of interest here is illustrated in fig. 2, which shows results currently available on the β-decay of ^{74}Rb [18,17]. Note, in particular, that the first-excited 2^+ state at 456 keV has an apparent net depopulation of 0.2%, yet it cannot possibly be populated to this extent by second-forbidden β-decay. The answer to this contradiction lies in the results of shell model calculations [19], which show that hundreds of 1^+ states are expected within the β-decay window, that in total they carry $\sim 0.9\%$ of the decay strength, but that the strongest individual branch carries $< 0.3\%$. Evidently, the full compliment of γ-ray feeding to the 456 keV state involves transitions too weak to have been observed. The calculations also indicate that half the decay strength from the 1^+ states goes directly to the ground state, bypassing the first-excited state. Thus the 456 keV transition is not a "collector" for all the decay strength from higher-lying states. To obtain a precise superallowed ft-value, one must either rely on such shell model calculations to correct for unobserved states or develop experimental techniques based on total absorption spectrometry.

Clearly, the Pandemonium effect will be the ultimate limit on the precision with which the superallowed branching ratios can be determined for these nuclei with $A \geq 62$. So far, ^{62}Ga and ^{74}Rb are the only two cases for which precise measurements are becoming available: in addition to the ^{74}Rb decay rates already mentioned, similar but detailed information is available on ^{62}Ga [20,21] and a precise half-life for ^{74}Rb was recently published [22]. Only time will tell how well the Gamow-Teller strength will be accounted for, and what precision will be achieved.

3 Conclusions

The contribution of superallowed β-decay to the apparent failure of CKM unitarity has stimulated considerable activity in improving and extending measurements and calculations. In spite of significant challenges, new results are beginning to appear and will likely lead to reduced uncertainties and a more definitive test of CKM unitarity.

This work was supported by the U.S. Department of Energy under Grant number DE-FG05-93ER40773 and by the Robert A. Welch Foundation.

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